Electro-Optical Frequency Shifting of Lasers
for Plasma Diagnostics

P. R. Forman +)

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# MAX-PLANCK-INSTITUT FÜR PLASMAPHYSIK

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Electro-Optical Frequency Shifting of Lasers for Plasma Diagnostics July 1977 (in English)

Abstract: An electro-optical frequency shifting device is proposed as an aid for plasma physics heterodyne interferometry and heterodyne scattering experiments. The method has the advantage over other electro-optic shifters, that a pure separable frequency shifted beam can be obtained even when less than half wave voltage is applied.

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#### Introduction

It is expected that heterodyne interferometry and scattering will play an increasing role in plasma diagnostics. If multiple beam interferometers are to be useful, methods of recording the signals automatically will have to be developed. The method proposed by Hugenholtz and Meddens /1/ employing a frequency shifted reference beam and a digital phase comparator would seem an extremely reasonable approach. The frequency shift is required to allow the direct electronic recording of the plasma produced phase shift. High frequency shifts ( ~ 100 MHz) are also required in the method of spacial interferometry by rapid beam sweeping currently under investigation by Kristal /2/. The methods of autocorrelation of intensity fluctuations could also benefit from heterodyne interferometry. The experiments of Dutt et al. /3/ can be viewed as a homodyne experiment where the reference beam is unshifted in frequency. If turbulence due to lower hybrid instabilities in typical theta pinch plasmas, for example, are to be studied by this means, detectors with hundreds of megahertz bandwidth will be required. It would seem more reasonable to shift the reference beam in frequency which would allow low frequency detection of high frequency phenomena. Similar considerations would apply to CO<sub>2</sub> heterodyne scattering experiment already performed on ATC Tokamak /4/.

It would be advantageous to have a simple device to shift the frequency of a laser to aid in such experiments. Of particular interest are the wavelengths 3.39 µm and 10.6 µm because excellent lasers are available at these wavelengths. The characteristics desired are as follows:

- 1.) Frequency shifts adjustable from DC to ~ 500 MHz
- 2.) Minimum or no adjustment of optics required when changing frequency

 Continuous operation to avoid any problem of timing of measurement and timing of pulsed experiments.

To date I know of only two methods which have been employed to produce a shifted beam that have actually proved useful on a plasma experiment. The first is the rotating mirror. The mirror is usually a corner mirror mounted on a turning wheel. Tens of megahertz can be produced in this way for 3.39  $\mu$ m /5/. It fulfills requirement No. 2 but it fails on Nos. 2 and 3. It has the further disadvantage that the exact frequency shift is unknown and must be measured either on a previous turn of the wheel /5/or simultaneously with a second interferometer designed to do this /1/. An additional problem is that the phase shift is often not a pure frequency shift. The frequency shift achieved by mechanically moving a mirror at a fixed velocity is proportional to  $\lambda^{-1}$ . Therefore high frequency operation is even more difficult at longer wavelengths.

The second method involves acousto-optic modulation of light /6/. This scheme essentially meets requirement No. 3, partially fulfills No. 1 and fails No. 2. The shifted beam is produced when the Bragg condition is fulfilled and hence an angular adjustment of the cell is required when there is an appreciable change in frequency. The acoustic power required to drive a given cell is proportional to  $\lambda^2$  where  $\lambda$  is the wavelength of the laser being used. Furthermore the damping of the acoustic waves is approximately proportional to the square of the driving frequency ( $\omega_m^2$ ). As a result it may well be difficult to obtain an acoustic cell capable of high frequency modulation of 10.6  $\mu$ m radiation. It is the purpose of this note to propose an alternate method which should have a high probability of success because it is a direct extension of results already achieved in the visible region.

#### Theory

The theory of the frequency shift using the transverse Pockels effect is developed in reference /7/. It is outlined here specifically for 43m crystals. This case is considered here because the development in reference /7/is sketchy and this class of symmetry will be of particular interest due to the availability of CdTe and GaAs crystals. These crystals are very transparent in the region of interest and have large unclamped electro-optic coefficients /8/.

	CdTe	GaAs
Refractive index n	2.6	3.3
n <sub>c</sub> r <sub>41</sub> m/V	$12 \pm 1 \times 10^{-11}$	$5.8 \times 10^{-11}$
Absorption coefficient		
a cm <sup>-1</sup>	0.005 ± 0.002	0.012 - 0.02
Transmission range	mu 30 – 1	mر 18 – 1

If a cube is viewed directly along a diagonal axis (z' in fig. 1) it has a three fold axis of symmetry. We consider light travelling along this axis subject to an electric field perpendicular to k. The electro-optic coefficients are not defined in this coordinate system so we define a transformation matrix which relates this system to the one in which the coefficients are normally defined.

Primed coordinate are lab, unprime crystal.

The transformation matrix between coordinate systems is

$$\widetilde{A} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 1 & -2 \\ -\sqrt{3} & \sqrt{3} & 0 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} \end{pmatrix} \qquad A_{ij} = A_{ji}$$

We assume an applied electric field in Lab coordinates of

In the crystal system this field is

The indicatrix equation is, for a 43m class crystal subject to this field

Converting this equation back to lab coordinates results in

$$\frac{x^{2}+y^{2}+z^{2}}{n_{c}^{2}}+\frac{2v_{4}}{\sqrt{6}}E_{m}\left[\cos x\left[y^{2}-x^{2}-\sqrt{2}z\right]+\sin x\left[2xy-\sqrt{2}yz\right]\right]=1$$

As long as light is going directly along the z axis, we must set z = 0 because the equation of interest is the intersection of the indicatrix with a plane normal to the vector of the light. We wish to eliminate the "xy" cross product term therefore consider an arbitrary rotation about the z axis.

$$\frac{x^{2}+y^{2}}{no^{2}} + \frac{2741}{\sqrt{6}} = m \left\{ (y^{2}-x^{2}) \left[ \cos x (\cos^{2}\phi - \sin^{2}\phi) - 2\sin x \cos \phi \sin \phi \right] + 2xy \left[ 2\cos x \sin \phi \cos \phi + \sin x (\cos^{2}\phi - \sin^{2}\phi) \right] = 1 \right\}$$

We require the coefficient of xy to be zero. Let  $\emptyset$  =  $\beta/2$ .

This requirement becomes

$$\cos \alpha \sin \beta = -\sin \alpha \cos \beta$$
or
$$\phi = -\frac{\alpha}{2}$$

In this rotated coordinate system the equation becomes

$$\frac{x^2 + y^2}{nc^2} + \frac{2r_{4i}}{\sqrt{6}} = m \left[ x^2 - y^2 \right] = 1$$

which is the equation of an ellipse.

The electric field of a left-handed polarized light beam in the lab system described in this rotated coordinate system is

After traversing the crystal apart from an overall phase shift that both components experience the emerging field will be

$$\vec{A} = A \left[ \cos(\omega t - \phi + \frac{\pi}{2}) \hat{e}_{x} + \sin(\omega t - \phi - \frac{\pi}{2}) \hat{e}_{y} \right]$$

$$\Gamma = 2\pi n_{o}^{3} l \frac{2\sqrt{4}}{\sqrt{6}} E_{m} / \frac{2n_{o}^{2} r_{o}^{2} E_{m}}{\sqrt{6}} \ll 1$$

Viewed in the original lab frame and assuming the applied electric field was a rotating field of frequency i.e.  $\ll = \omega_m t$ 

which is our original left-circularly polarized wave reduced in amplitude by the factor  $\cos(7/2)$  plus a right-circularly polarized wave with a shifted frequency.

It has been assumed the transit time through the crystal is small compared to

This restriction could be overcome by a series of crystals which either have angularly displaced fields or are driven with an appropriate phase shift so as to approximate a travelling wave.

If either direction of rotation were reversed the results would be the same except the shift would now be a down shift. The shifted beam will always have the polarization opposite to that of the incident beam.

As pointed out by Buhrer et al. crystals of the 23, 43 m, 6, 6m2, 3, 32 and 3 m groups could be used to produce a modulator. This technique has the advantage over other electro-optical shifters /9/, in that it can produce a separable, pure frequency shifted laser beam at low applied voltage. This feature could be especially convenient for plasma heterodyne scattering experiments where only extremely weak local oscillators are required.

It was assumed that equal amplitude fields were applied in the x and y directions with the proper  $90^{\circ}$  phase shift. In a real device this will probably not be the case. We assume that the applied field really has the form

The condition required to eliminate cross product terms then becomes

Inspection of this equation shows that it is only soluble for the case  $\mathcal{E} = \mathcal{S} = 0$  which means the equation is no longer elliptical so other frequencies will be produced.

We consider the two errors separately. The most probable error is that the strength of the applied fields will be unequal. Such a field can be resolved into two separate fields

$$\vec{E} = E_m (1 - \frac{6}{2}) \left[ \cos \alpha \hat{e}_x + \sin \alpha \hat{e}_y \right]$$

$$+ E_m \frac{\epsilon}{2} \left[ \cos \alpha \hat{e}_x - \sin \alpha \hat{e}_y \right]$$

which represents a reduced left circularly-polarized field plus a right circularly-polarized error field. Assuming these fields act independently, the result of such an error will be the introduction of a small component of frequency  $\omega - \omega_m$  in the shifted beam. This will produce, in addition to the optical frequency component which would appear as DC to a photodetector, a radio frequency component at twice the applied frequency.

$$I_{rf} \sim I_c e^3 7^2 \cos(2 \cos t)$$
 ( $I_c$  is the incident intensity of the laser)

Similar considerations apply to an error in phasing. A phase error can be expressed, for small  $\delta$  , as

$$\vec{E} = E_m \left[ \cos \alpha \, \hat{e}_x + \sin \alpha \, \hat{e}_y \right]$$

$$+ E_m \frac{1}{2} \left[ \cos \left( \alpha + \frac{\pi}{2} \right) \hat{e}_x + \sin \left( \alpha + \frac{\pi}{2} \right) \hat{e}_y \right]$$

$$- E_m \frac{1}{2} \left[ \cos \left( \alpha + \frac{\pi}{2} \right) \hat{e}_x - \sin \left( \alpha + \frac{\pi}{2} \right) \hat{e}_x \right]$$

which also leads to an oscillating term

The absence of such components in the shifted beam would be taken as evidence of proper adjustment of these quantities. If elliptically-polarized light were used instead of circularly-polarized light, a radio frequency component

$$I_{tf} \sim I_o \in \sin^2 l_2 \cos(2\omega_m t)$$
 would be introduced into the "unshifted" beam.

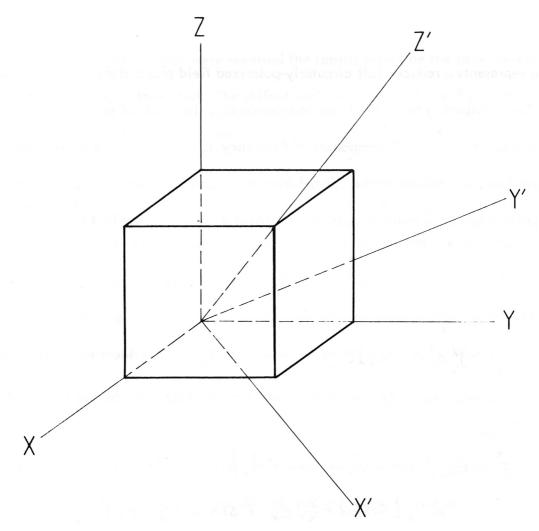


Fig. 1: Coordinate systems: primed coordinates are lab coordinates; unprimed coincide with the crystal axies

## Application

Such a device has the possibility to be directly applicable to a number of plasma interferometer or heterodyne scattering experiments. As an example, a schematic drawing of a heterodyne Mach-Zehnder interferometer is shown in Fig. 2. The output of a linearly polarized laser is first converted to circularly polarized light by passing through a quarter-wave plate. For the wavelength considered here (3.39 and 10.6 µm) a Fresnel rhomb /9/ might well be more convenient than an actual quarter wave plate. After passing through the optical shifter, a second quarter wave plate converts the circularly polarized waves back to linearly polarized waves. A polarization sensitive

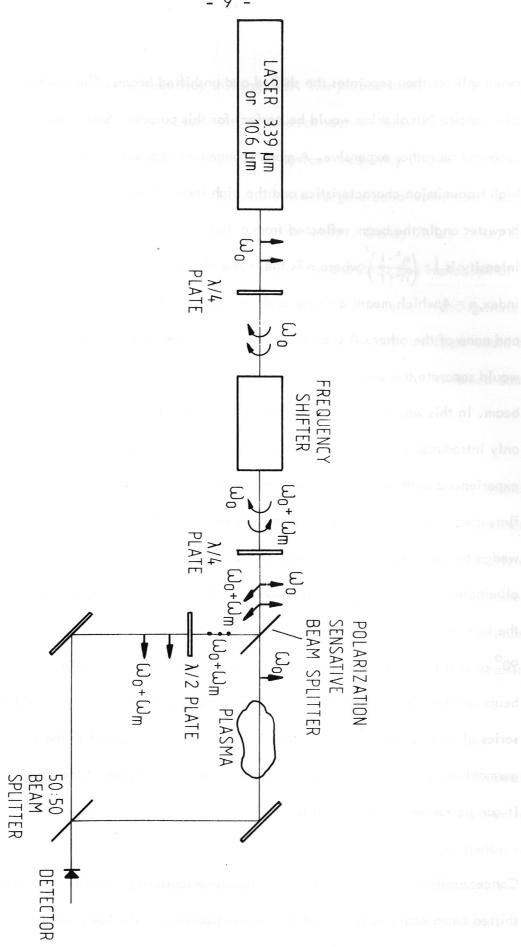


Fig. 2: Schematic drawing of a heterodyne Mach-Zehnder interferometer

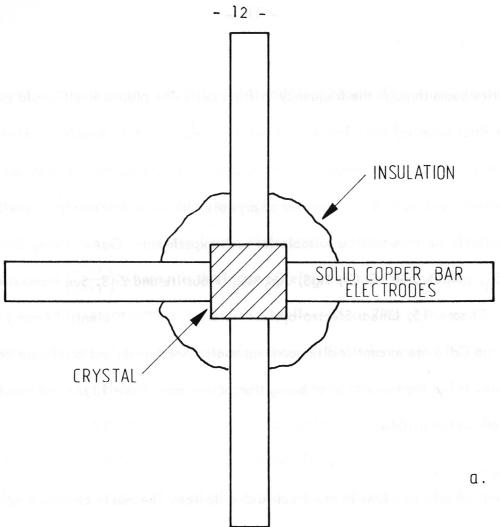
beam splitter then separates the shifted and unshifted beams. The infrared equivalent of a calcite Nicol prims would be perfect for this purpose. Such a device would probably be rather expensive. A more economical approach would be to exploit the high transmission characteristics and the high index of refraction of Germanium. At Brewster angle the beam reflected from a dielectric is entirely polarized, and its intensity is  $\hat{L} = \left(\frac{n^2-1}{n^2+1}\right)^2$  where n is the index of refraction. Germanium has a very large index, n = 4, which means a single surface will reflect about 78% of the proper polarization and none of the other. A second germanium Brewster flat orientated at 90° to the first would separate the other orientation from the remaining component in the transmitted beam. In this way a polarization sensitive beam splitter could easily be constructed which only introduces about 22% loss which is comparable to the loss due to absorption experienced with partially silvered mirrors that are used in the visible. The germanium flats used in this Brewster's angle beam splitter should not have parallel faces. A slight wedge shape ~10 mrad will insure that reflections from the second surface will be eliminated by total internal reflection. The last component which is necessary to make the interferometer work is to rotate the plane of polarization of the reference beam by 90° so that it will interfere with the scene beam when it is recombined on the 50: 50 beam splitter. This is indicated as a  $\lambda/2$  plate in the schematic. It could in fact be a series of mirrors which leave the plane of the paper to accomplish the same effect. The germanium beam splitter suggested above sends one beam out of the main plane already. It can be rotated in bringing it back.

Conceptually the arrangement for a heterodyne scattering experiment is similar. The shifted beam could be produced from a small portion of the laser beam to avoid passing

the entire beam through the frequency shifting cell. The plasma itself would substitute for the final beam splitter. The components to produce such an interferometer are then easily available. It only remains to construct an optical-frequency shifter, assemble the elements and try it out. A number of crystals with large electro-optic coefficients are available which would be suitable for such experiments. GaAs, ZnTe, ZnSe, CdTe, LiNbO<sub>3</sub>, LiTaO<sub>3</sub>, Se, CuCl, HgS, Ag<sub>3</sub>AsS<sub>3</sub> (Proustite) and ZnS. See Handbook of Lasers, Chapter 15, Linear Electro-optical materials for Coefficients). Among these GaAs and CdTe are exceptionally good materials. Another serious candidate could be Proustite. It has the advantage of being transparent from 0.6 - 13 µm and hence could also work in the visible.

#### Construction

The question arises on how to construct such a device. The most obvious way would be to follow the example of Buhrer et al. and simulate a rotating field by four electrodes attached to the surfaces of a bar crystal (fig. 3a) and drive the adjacent electrodes with the 90° phase shift. These electrodes would have to be smaller than the surface area to allow for voltage insulation between adjacent electrodes. The space between adjacent electrodes would be filled with a good high frequency insulator (for example paraffin) to help avoid flash over. The electrodes should be solid copper bars to help damp acoustic oscillations that will occur. Larger cross section crystals would require proportionally higher voltages but would provide a more uniform electric field assuming that a constant cross section laser beam is used. Another possible solution might be to give up the damping provided by the electrodes and employ an arrangement similar to that in fig. 3b. In this arrangement the crystal is kept small but the electrodes are removed. The choice of configuration would require experimentation to determine if



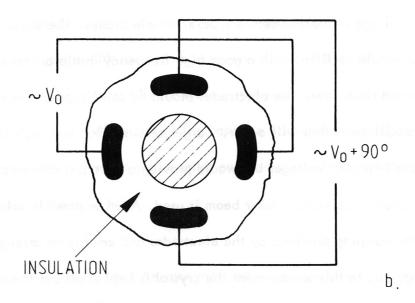


Fig. 3: Two possible electrode configurations

acoustic oscillations are serious. Provisions should be made in the power supply to provide an error in the amplitude or phasing of the applied voltage as an aid in allignment of the polarization sensitive beam splitter.

For a numerical example we consider a CdTd crystal. For an interferometer the polarization sensitive element is acting as a beam splitter. At most we will require this to be a 50:50 division. This then sets a maximum on the required applied field.

assuming a crystal 5 cm long, this requires a field of

With direct contact and a 3 mm wide crystal this would give 5.2 and 16.2 kV for 3.39 and 10.6 µm which are rather high voltages but might still be possible. There are two saving features: 1) a 50 : 50 division is in all probability not desirable. The scene beam of an interferometer usually suffers considerably more loss of intensity than does the reference beam. This is due to the extra reflections at vacuum vessels etc. 2) The second saving feature is that in an interferometer the electric field add rather than the intensities.

Therefore  $\overrightarrow{A} = \overrightarrow{A}_1 + \overrightarrow{A}_2 = A_1(1 + 8e^{(4)})$   $I \sim AA^* = I[1 + 8^* + 28\cos\phi]$ 

where  $\gamma$  is a constant of proportionality and  $\varnothing$  is some phase difference.

We can define a fringe contrast ratio

$$FC = \frac{28^2}{1+8^2}$$

If we are satisfied with a fringe contrast of 50%, 3 can be as small as 0.27. As a result of these two facts, it is expected that a modulator with only a tenth of the equal

division voltage would still be a very useful device.

If simple contact electrodes are employed the frequency shifter would then be CdTd crystal 3 mm x 50 mm cut with the end faces to be [111] planes so the light propagates along the 3-fold axis. These ends would be antireflection coated to avoid excessive losses because of the high index of refraction of CdTd. Four electrodes ~ 1.5 mm wide are plated on the surfaces. One and a half mm wide copper bars are used to attach to these electrodes. These bars would help to damp out acoustic waves which will be generated. The surface of the crystal between these electrodes is covered with a good high frequency insulator (such as paraffin) to avoid surface flashover. The opposite pairs of electrodes are driven with high voltage generators with the appropriate 90° phase shift.

### Two-Dimensional Interferometry

An intriguing question is whether these crystals will also exhibit this frequency shift behavior when the applied field is in the optical rather than the radio range. Proustite has shown such shifts under phase matching conditions /11/. If this is the case the possibility exists to shift infra red lasers into the visible where detection of two-dimensional interferograms would be greatly facilitated. Because of the vastly greater sensitivity of photographic film vis a vis the energy detectors which until now have been employed /12/, only a very small amount of the light would have to be converted. This is even more true when one considers detection with a vidicon as opposed to film. Modern optical vidicons with resolutions of 80 line pair / mm allow one to consider an electronic analogue to 2-D holographic interferometry.

For reliable operation one would want to limit the total length of a crystal such that  $\Delta N_o \stackrel{Q}{\nearrow} \stackrel{1}{\searrow} \stackrel{1}{\swarrow} \frac{1}{4}$  where  $\Delta N_c$ , the difference in index of refraction of proustite for the two waves of interest, is used because both waves are travelling through the crystal. To estimate this length we use Sellmeier dispersion equation

$$n_0^2 = A + \frac{B_1}{\lambda^2 - B_2} - \frac{C_1}{C_2 - \lambda^2}$$

where \(\) is expressed in \(\mu\mathrm{m}\).

$$A = 9.220$$
  $B_1 = 0.4454$   $B_2 = 0.1264$   $C_1 = 1733$   $C_2 = 1000$  for Proustite.

Choosing 10.6 µm and 1.06 µm as an example  $\triangle n_e = 0.12$  or  $\ell = 7$  µm microns thick which is rather thin. The production of such a thin crystal sheet would seem to be the limiting factor. The damage threshold for proustite is high enough that sufficient light could be converted /13/.

## Increased Sensitivity for Heterodyne Interferometry

The phase shift that a plasma produces in an interferometer has generally been recorded by one of two ways. Either the amplitude of the emerging recombined beams is measured as a function of time and is interpreted as a phase shift, or a "moving mirror" producing a doppler shift with a higher frequency than the event of interest is introduced and the time of zero crossing of the signals is recorded. Both methods have their advantages which leads one to choose one method over the other in a given situation. One that the amplitude method possesses is the ability to record very small phase shifts. If the signal is too small, often it is simple to amplify it. If sufficiently reliable and stable

electro-optical phase shifters are in fact achievable, this "amplification" may also be achievable with heterodyne interferometers. The output signal from a heterodyne interferometer is of the form

where  $\omega_c$  is the heterodyne frequency and  $\Theta(t)$  is the phase shift introduced by the plasma. "Amplification" for a heterodyne interferometer implies amplification of the argument of the cosine function. For an amplitude sensitive measurement the coefficient A would be multiplied. In both cases it is assumed the desired signal is not obscured; it is just too small to measure accurately so amplification will help.

To amplify the frequency ( $\omega_{\mathbf{c}} t + \mathcal{C}(t)$ ) we will wish to generate harmonics. To do this, we process the signal by a suitable non-linear amplifier. The output would appear, in frequency space, something like that of Fig. 4. The amplitudes of the various harmonics would depend on the specific non-linear device chosen. In drawing Fig. 4, it has been assumed that the Fourier components of  $\Theta(t)$  are all very much less than  $\omega_{\mathbf{c}}$ . The output of the non-linear device is then filtered to eliminate all but one harmonic. Our signal is now approximately of the form  $V \sim A \cos(N\omega_c \dot{t} + N\Theta(t))$  where N is the harmonic chosen (N = 4 in Fig. 4). If this signal is heterodyned with a local oscillator  $\dot{\omega}_H = (N-1)\omega_c$  the output voltage will be  $V = A\cos(\omega_c \dot{t} + N\Theta(t))$  resulting a net amplification of the phase shift of interest. By introducing the bandpass filter centered on the N harmonic, we have introduced some errors because the Fourier integral goes over all frequencies. The allowable bandwidth of the bandpass filter is the carrier frequency  $\omega_c$ . We require that no fourier components of the adjacent harmonics are within the bandpass of this filter. This means the input, or detector frequency bandpass

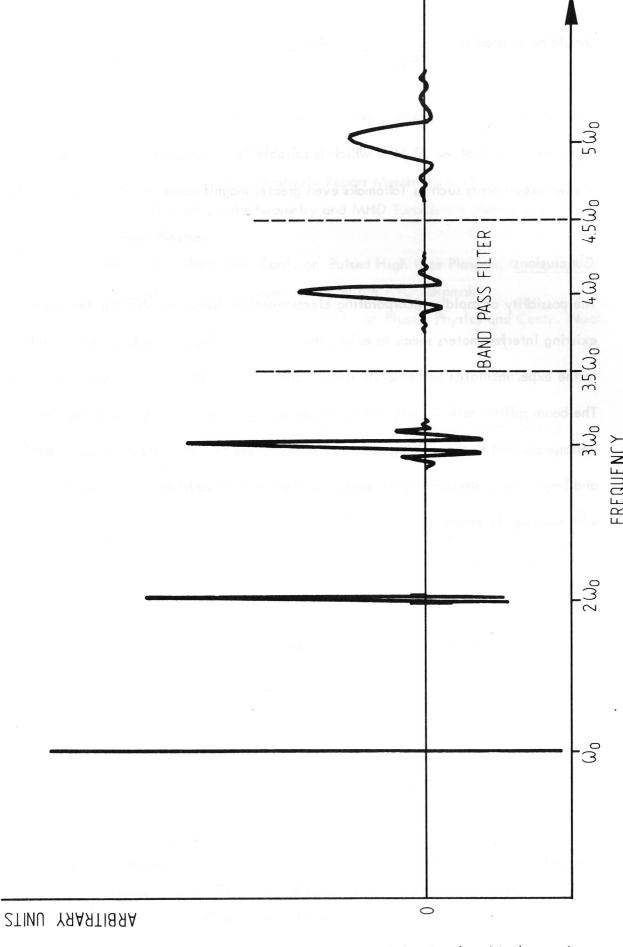


Fig. 4: Schematic Fourier representation of the signal and its harmonics, from a heterodyne interferometer

 $\omega_{DET} = \frac{\omega_c}{2(N+1)}$ should be limited to

Assuming a 500 MHz frequency shift a factor of ten magnification would allow a signal bandwidth of ~ 23 MHz which is suitable for many applications. With slower experiments such as Tokamaks even greater magnification would seem possible.

#### Conclusions

The possibility of rapidly incorporating electro-optical frequency shifting devices into existing interferometers seems to exist. The frequency of operation is a variable left to the experimentalist and possibly is even changeable during a single plasma discharge. The beam splitter ratio is selectable, within voltage limitations, by adjusting the voltage applied to the shifting element. With the use of such devices the autocorrelation and homodyne scattering experiments could be extended to higher frequency domains with existing detectors.

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